



RIMS Workshop

Automorphic forms, automorphic representations and related topics

Abstract

Jan. 21 (Mon.)

13:30 – 14:30 **Masao Tsuzuki** (Sophia University)

Title: Explicit trace formula of $SL_3(\mathbb{Z})$ and its application.

Abstract:

14:45 – 15:45 **Shingo Sugiyama** (Nihon University)

Title: A variant of trace formula related with triple products of modular forms

Abstract: Zagier gave a formula of an average of symmetric square L-functions associated with holomorphic elliptic cusp forms with complex parameter z , whose specialization at $z = 1$ recovers the Eichler-Selberg trace formula. His proof uses real analytic Eisenstein series and unfolding of integrals. In this talk, we give an analogous formula of Zagier's one by replacing real analytic Eisenstein series with a Maass cusp form and by representation theory. As an application of our formula, for a fixed Maass cusp form f on $GL(2)$, we give an infinitude of cohomological automorphic forms F on $GL(3)$ such that $L(1/2, f \times F)$ is non-vanishing. This is a joint work with Masao Tsuzuki (Sophia University).

16:00 – 17:00 **Hiraku Atobe** (Hokkaido University)

Title: Liftings of vector valued Siegel modular forms (Survey Talk).

Abstract: This is a survey talk on liftings of Siegel modular forms. Chenevier-Lannes gave a general description of the space of vector valued Siegel modular cusp forms by using Arthur's multiplicity formula. In particular, this description tells us the existence of many liftings of cusp forms. In this talk, I will explain the description of Chenevier-Lannes, and give several examples of liftings, which contain Ibukiyama's conjecture.

Jan. 22 (Tue.)

9:30 – 10:30 **Hiroataka Kodama** (Kogakuin University)

Title: On certain vector valued Siegel modular forms of type $(k, 2)$ over $\mathbb{Z}_{(p)}$ and Sturm bound.

Abstract: Let p be a prime number and $\mathbb{Z}_{(p)}$ is the local ring of the p -integral rational numbers. In this talk we give generators over the graded ring over $\mathbb{Z}_{(p)}$ of the Siegel modular forms of degree 2 with even weight whose Fourier coefficients lie in $\mathbb{Z}_{(p)}$ of its module for an even integer k when p is larger than or equal to 5. In addition, we talk about the Sturm bound of the vector valued Siegel modular forms of type $(k, 2)$ and degree 2.

10:45 – 11:45 **Shuji Horinaga** (Kyoto University)

Title: On constructions of nearly holomorphic Siegel modular forms.

Abstract: By Pitale-Saha-Schmidt, there exists a classification of $(\mathfrak{g}, \mathbb{K})$ -modules generated by nearly holomorphic modular forms of degree 1 and 2. In order to determine the multiplicities, we construct nearly holomorphic modular forms via Klingen Eisenstein series and Rankin-Cohen bracket. In this talk, we discuss the constructions and these generalizations.

13:30 – 14:30 **Niko Laaksonen** (Alfréd Rényi Institute of Mathematics)

Title: Prime Geodesic Theorem in the three-dimensional hyperbolic space.

Abstract: The Prime Geodesic Theorem (PGT) states that the lengths of primitive closed geodesics on a hyperbolic manifold have an asymptotic behaviour analogous to the usual prime numbers. Through an explicit formula for the Selberg zeta function we can relate the error term to certain spectral exponential sums. In the past few years there has been a renewed interest in this problem especially in two and three dimensions. In this talk we will outline some recent progress on the pointwise and second moment bounds of the error term in the PGT on various three dimensional hyperbolic manifolds.

14:45 – 15:45 **Pablo Ramacher** (Universität Marburg)

Title: The equivariant spectral function and non-spherical subconvex bounds for Hecke-Maass forms.

Abstract:

16:00 – 17:00 **Kazuki Kannaka** (University of Tokyo)

Title: On the discrete spectrum of a certain non-sharp locally anti-de Sitter space.

Abstract: The discrete spectrum (= the set of L^2 -eigenvalues) of the Laplace-Beltrami operators on Riemannian locally symmetric spaces $\Gamma \backslash G/K$ has been investigated extensively. However, this is not the case for non-Riemannian locally symmetric spaces $\Gamma \backslash G/H$. Kassel-Kobayashi [Adv. Math., 2016] initiated the study of analysis in these cases and constructed “universal discrete spectrum” under some conditions (for example, “the sharpness” of Γ). One of the key ideas to construct infinitely many eigenvalues on $\Gamma \backslash G/H$ is “counting Γ -orbits” in G/H . Indeed, they established upper exponential bounds for the growth of orbits of “sharp” discontinuous groups.

The aim of this talk is to construct a “non-sharp” example to which their method is still applied. For this purpose, we generalize the argument in Guéritaud-Kassel [Geom. topol., 2017] and construct a family of infinitely generated groups $\Gamma_N^{j,\rho}$, which act isometrically and properly discontinuously on the 3-dimensional anti-de Sitter space AdS^3 . We prove that these actions are “non-sharp” in the sense of Kassel-Kobayashi and the discrete spectrum of $\Gamma_N^{j,\rho} \backslash \text{AdS}^3$ for an appropriate choice of j, ρ , and N is infinite. As a biproduct, we prove that for “any function” $f(R)$, there exist j, ρ and N such that the number of points of a certain $\Gamma_N^{j,\rho}$ -orbit in “the pseudo-balls” of radius R is larger than $f(R)$ for sufficiently large R .

Jan. 23 (Wed.)

9:30 – 10:30 **Toshiki Matsusaka** (Kyushu University)

Title: Fourier coefficients of polyharmonic weak Maass forms.

Abstract: Higher Laurent coefficients of the real analytic Eisenstein series at $s=1$ are annihilated by a number of iterations of the action of the hyperbolic Laplace operator. Focusing on this property, Lagarias and Rhoades introduced a new space of polyharmonic Maass forms in 2016. This gives an extension of generalized mock modular forms appeared in Duke-Imamoglu-Toth’s work on traces of cycle integrals of the j function. In this talk, we introduce polyharmonic weak Maass forms more generally, and extend their results. As a corollary, we give a new perspective to some classical results.

10:45 – 11:45 **Kohta Gejima** (Osaka University),

Tomonori Moriyama (Osaka University)

Title: On an average of critical values of Rankin–Selberg L -functions.

Abstract: For any even integer n , we denote by \mathcal{S}_n the space of cusp forms of weight n for $SL_2(\mathbb{Z})$. Let k, l be nonnegative even integers with $k > l$. For a fixed $g \in \mathcal{S}_l$, we give an explicit expression for the average of special values of the Rankin–Selberg L -function $D(s, f \otimes g)$ at $s = \frac{k+l}{2}$ as f ranges thorough an orthogonal basis of \mathcal{S}_k . This supplements an earlier result of Lanphier on the other critical values. This is a joint work with Akimitsu Ohta and Masaki Yagi.

13:30 – 14:30 **Werner Hoffmann** (Universität Bielefeld)

Title: On the Fourier transforms of weighted orbital integrals on the real symplectic group of rank three.

Abstract:

14:45 – 15:45 **Werner Müller** (Universität Bonn)

Title: Asymptotic distribution of automorphic spectra and the trace formula.

Abstract: For a given lattice in a semisimple Lie G we consider the discrete spectrum of the right regular representation of G in L^2 of the corresponding coset space. In this talk I will discuss the behavior of the discrete spectrum with respect to the growth of various parameters such as the level of congruence subgroups or/and the infinitesimal character. This is closely related to the study of families of automorphic forms in the sense of Sarnak. A number of results are known for $GL(n)$ and the issue is to extend these results to other groups. The main technical tool is the Arthur trace formula.

16:00 – 17:00 **Siegfried Böcherer** (Universität Mannheim)

Title: Nonvanishing properties of Fourier coefficients for Siegel modular forms.

Abstract: We report on joint work with S.Das. We show that for a Siegel modular form there are infinitely many nonvanishing Fourier coefficients indexed by quadratic forms of squarefree discriminant (if the weight is not too small). We also discuss variants and generalizations of this statement. The case of degree two is a result of Saha.

Jan. 24 (Thu.)

9:30 – 10:30 **Tomoaki Nakaya** (Kyushu University)

Title: The number of linear factors of supersingular polynomials and sporadic simple groups.

Abstract: The set of prime numbers p such that the supersingular j -invariants in characteristic p are all contained in the prime field is finite. And it is well known that this set of primes coincides with the set of prime divisors of the order of the Monster simple group. In this talk, I will present analogous coincidence of supersingular invariants in level 2 and 3 and the orders of the Baby monster group and the Fischer's group. The proof uses a connection between the number of supersingular invariants and class numbers of imaginary quadratic fields.

10:45 – 11:45 **Jiangwei Xue** (Wuhan University)

Title: On counting certain principally polarized superspecial abelian surfaces over \mathbb{F}_p .

Abstract: We study the principally polarized superspecial abelian surfaces over the prime finite field \mathbb{F}_p with Frobenius endomorphism $\pi = \pm\sqrt{p}$. We give a description of this set in terms of double coset spaces and attempt to obtain an explicit formula for its cardinality. This is a joint work with Prof. Chia-Fu Yu.

13:30 – 14:30 **Nadir Matringe** (Université de Poitiers)

Title: Deligne representations of the Weil group modulo l .

Abstract: Let F be a non archimedean local field of characteristic p , W_F its Weil group, and let R be the algebraic closure of F_l . If Φ is a finite dimensional semi-simple representation of W_F with coefficients in R , and U is an endomorphism of the space of Φ satisfying the Deligne condition, then U is not necessarily nilpotent (unlike when $R = C$). We classify isomorphism classes of such representations (Φ, U) up to irreducible representations of W_F , as well as equivalence classes of such representations (where in the indecomposable case (Φ', U') is equivalent to (Φ, U) if there is a nonzero r in R such that (Φ', U') is isomorphic to (Φ, rU)).

One can define local constants of such (equivalence classes of) Deligne representations as usual, and we construct an injection of the set of isomorphism classes of l -modular irreducible representations of $GL(n, F)$ into the set of equivalence classes of n -dimensional Deligne representations (Φ, U) , which makes the local constants on the GL side match with the local constants on the Galois side in the generic case.

14:45 – 15:45 **Alexei Pantchichkine** (Institut Fourier, University Grenoble-Alpes)

Title: A motivic approach to Shimura's zeta functions and attached p -adic L -functions via admissible measures.

Abstract: A motivic approach is presented to Shimura's zeta functions $\mathcal{Z}(s, \mathbf{f})$ [Shi00] attached to nearly holomorphic arithmetical automorphic forms \mathbf{f} on unitary groups U_K of an imaginary quadratic field K . A motivically normalized L -function $\mathcal{D}(s, \mathbf{f})$ attached to $\mathcal{Z}(s, \mathbf{f})$ is defined in accordance with Deligne [De79] and Coates-Perrin-Riou conjectures [CoPe]. An explicit description of Shimura's Γ -factors is used.

The attached p -adic L -functions of $\mathcal{D}(s, \mathbf{f})$ are constructed via admissible measures.

The main result is stated in terms of the Hodge polygon $P_H(t) : [0, d] \rightarrow \mathbb{R}$ and the Newton polygon $P_N(t) = P_{N,p}(t) : [0, d] \rightarrow \mathbb{R}$ of the zeta function $\mathcal{D}(s, \mathbf{f})$ of degree $d = 4n$. Main theorem gives a p -adic analytic interpolation of the L values in the form of certain integrals with respect to Mazur-type measures.

Both Rankin-Selberg and doubling methods are used

[CoPe] COATES, J., PERRIN-RIOU, B., *On p -adic L -functions Attached to Motives over \mathbb{Q}* . Advanced Studies in Pure Mathematics 17, 1989 Algebraic Number Theory in honor of K. Iwasawa pp. 23-54.

[De79] DELIGNE P., *Valeurs de fonctions L et périodes d'intégrales*, Proc. Sympos. Pure Math. vol. 55. Amer. Math. Soc., Providence, RI, 1979 , 313-346.

[Shi00] SHIMURA G., *Arithmeticity in the theory of automorphic forms*, Mathematical Surveys and Monographs, vol. 82 (Amer. Math. Soc., Providence, 2000).

16:00 – 17:00 **Jaeho Haan** (KIAS)

Title: Fourier-Jacobi period and central L -value.

Abstract: The Gan-Gross-Prasad(GGP) conjecture predicts that the non-vanishing of certain periods is equivalent to the non-vanishing of certain central value of some L -function. There are two types of periods: Bessel period, Fourier-Jacobi(FJ) period. Bessel period is period of automorphic forms on orthogonal group or hermitian unitary group and FJ period is period on symplectic and metaplectic group or skew hermitian group. For unitary group, Bessel case was proved by Wei Zhang and FJ-case was also by Hang Xue. But both results are under certain local restriction to apply relative trace formula.

In 2014, Ichino and Yamana proved one direction of Bessel case for unitary group and Yamana prove one direction of FJ case for symplectic and metaplectic group. Since they didn't use relative trace formula, their results are without local restrictions.

Following Yamana's approach, we prove one direction of the FJ case for skew-hermitian unitary group. This is a joint work with Hiraku Atobe.

Jan. 25 (Fri.)

9:10 – 10:10 **Yuya Murakami** (Tohoku University)

Title: Intersection numbers of modular correspondences.

Abstract: A modular polynomial is a symmetric polynomial of two variables with integral coefficients characterized by the j -invariant function. This polynomial defines an affine plane model over \mathbb{Z} of a certain modular curve. Hurwitz gave a formula of the intersection number of two plane algebraic curves defined by two modular polynomials. In this talk, I will introduce modular polynomials characterized by Hauptmodul's of certain modular curves of genus zero and give an analogue formula of the intersection numbers which generalizes the Hurwitz's formula.

10:20 – 11:20 **Henry Kim** (University of Toronto)

Title: Equidistribution theorems for holomorphic Siegel cusp forms.

Abstract: We explain equidistribution theorems for a family of holomorphic Siegel cusp forms of GSp_4 in the level and weight aspects. A main tool is Arthur's invariant trace formula. While Shin-Templier used Euler-Poincare functions at the infinity in the formula, we use pseudo-coefficients of holomorphic discrete series to extract only holomorphic Siegel cusp forms. Then the non-semisimple contributions arise from the geometric side, and this provides new second main terms which have not been studied, and which correspond to endoscopic cuspidal representations with large discrete series at the infinity. We give several applications, including the vertical Sato-Tate theorem and low-lying zeros for degree 4 spinor L-functions and degree 5 standard L-functions of holomorphic Siegel cusp forms. This is a joint work with Satoshi Wakatsuki and Takuya Yamauchi. If time permits, we explain work in progress to the generalization to Sp_{2n} .

11:30 – 12:30 **Shunsuke Yamana** (Kyoto University)

Title: Towards the p -adic Gross-Zagier formula for triple product L -series.

Abstract: Benedict Gross and Stephen Kudla conjectured a relation between central derivatives of triple product L -series of a triple of elliptic curves and the Beilinson height of diagonal cycles. This conjecture was largely proved by Xinyi Yuan, Shou-Wu Zhang and Wei Zhang. I will give a precise formulation of its p -adic analogue, which relates cyclotomic central derivatives of p -adic triple product L -series to Nekovar's p -adic heights of diagonal cycles. This is a joint work with Ming-Lun Hsieh.